# History based flow analysis in the lambda calculus

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History-based stack inspection





Labeled lambda-calculus



Many calculi exist since [76, Denning's]:

- [97 Biswas], [97 Abadi, Lampson, JJL] dependency calculus for *makefiles*
- [98-00 Pottier, Simonet, Heintze, Riecke] type theory with security information à la
   [97 Volpano, Smith] for ML-like programs.
- [99 Abadi, Banerjee, Heintze, Riecke] Dependency core calculus
- [00 Boudol, Castellani] Imperative programs
- ... type checking + type inference

Non interference theorems.

#### Non interference



- *M* public (low), *A* is private (high)
- $M \rightarrow V$ , V value
- no leak of A in V

- All (but first) are based on type theory and non-interference.
- Is there an "untyped" theory ?
- Is non-interference wrt "security levels" the only property?

# Stack inspection (1/5)

#### [Fournet, Gordon, POPL'02]

- flow analysis based on procedure calls
- JVM + CLR security manager  $\Rightarrow$  stack inspection

Stack inspection supports two sets of permissions:

- dynamic permissions D
- static permissions S
- reduction  $\longrightarrow_{D}^{S}$  is parameterized by *D* and *S*

# Stack inspection (2/5)

Language

$$R, S, D ::=$$
permissions set $M, N ::=$ expression $x \mid \lambda x.M \mid MN$  $\lambda$ -expression $R[M]$ framed expressiongrant  $R$  in  $M$ permission granttest  $R$  then  $M$  else  $N$ permission test

$$V \qquad ::= \lambda \mathbf{X}.\mathbf{M}$$

value

Reductions

call-by-value

$$\frac{M_1 \longrightarrow_D^S M_1'}{M_1 M_2 \longrightarrow_D^S M_1' M_2}$$

$$\frac{M_2 \longrightarrow_D^S M_2'}{V_1 M_2 \longrightarrow_D^S V_1 M_2}$$

 $(\lambda \mathbf{x}.\mathbf{M})\mathbf{V}\longrightarrow_{\mathbf{D}}^{\mathbf{S}}\mathbf{M}\{\mathbf{x}:=\mathbf{V}\}$ 

# Stack inspection (3/5)

• permission rules [CtxFrame]

$$\frac{M \longrightarrow_{D \cap R}^{R} M'}{R[M] \longrightarrow_{D}^{S} R[M']}$$

$$\frac{[CtxGrant]}{M \longrightarrow_{D \cup (R \cap S)}^{S} M'}$$
grant R in  $M \longrightarrow_{D}^{S}$  grant R in  $M'$ 

 $[RedFrame] \\ R[V] \longrightarrow_D^S V$ 

 $[RedGrant] \\ grant R in V \longrightarrow_D^S V$ 

#### [RedTest]

test R then  $M_{true}$  else  $N_{false} \longrightarrow^S_D M_{R \subseteq D}$ 

- $\cup$ ,  $\cap$ ,  $\subseteq$  are operations on permissions
- values are transparent for permissions
- static permission does not propagate in framed expressions
- stack inspection is a simple "untyped" calculus

## Stack inspection (4/5)

Example with Java-like programs

```
public static void main (String[ ] args) {
  NaiveLibrary.cleanUp ("/etc/passwd");
} }
public class NaiveLibrary { // ---- trusted
 static void cleanUp (String s) {
  File.delete (s);
} }
public class File { // -----trusted
 static void delete (String s) {
  FileIOPermission p = new FileIOPermission(s);
  p.checkDelete();
  System.deleteFile(s);
} }
```

 check fails with stack inspection since Applet[main(Lib[cleanUp(Sys[test FileDelete in delete(s) else fail])])] Applet ∩ Sys = ∅

- stack inspection provides a weak non-interference property
- → static analyzer for C# libraries
   [04, Blanc, Fournet, Gordon]
- with long proofs for soundness

## History-based stack inspection (1/2)

• [03, Abadi, Fournet] informal description of history-based stack inspection solving 2 examples:

```
■ BadPlugin example ↔ untrusted values
 class NaiveProgram { // -----trusted
   public static void main (String[ ] args) {
    String s = BadPlugin.tempFile ();
    NaiveLibrary.cleanUp (s);
  } }
 public class NaiveLibrary { // ----- trusted
   static void cleanUp (String s) {
    File.delete (s);
 static String tempFile () {
    return "/etc/passwd";
```

 does not fit in stack inspection since values are transparent for permissions

#### History-based stack inspection (2/2)

 Chinese Wall: B should not access to private information of A and conversely

```
public class Customer {
    int examine () {
      . . .
     if (shouldConsiderA) {
       Contractor a = new companyA();
       return a.offer();
    static public void main (String[ ] args) {
     int offer = examine ();
     Contractor b = new companyB();
     // ———raises exception if any B code has run
  } }
does not fit in stack inspection
  since not in a chain of function calls
```

#### Non interference between sub-expressions



- A and B are two different parties
- $M \rightarrow V$ , V value
- no interaction between A and B is necessary to produce V
- V may contain A and B
- interference theorem much harder to state

What is interaction between A and B?

#### Dependency calculi and Confluency

- confluency ≡ independence of evaluation strategy
   ⇒ equational theory ⇒ simplicity
- confluency ⇒ static analysis by abstract interpretation
- dynamic information is inherently non confluent as for the dynamicaly-scoped λ-calculus

$$(\lambda x.\lambda y.(\lambda x.\lambda y.x)yx)ab \longrightarrow \ldots \longrightarrow (\lambda x.\lambda y.x)ba \longrightarrow a$$
  
 $(\lambda x.\lambda y.(\lambda x.\lambda y.x)yx)ab \longrightarrow \ldots \longrightarrow (\lambda x.\lambda y.y)ab \longrightarrow b$ 

stack inspection is not confluent

when  $FileIO \subseteq Sys$   $Sys[(\lambda x. Applet[x]V)(\texttt{test} FileIO in (\lambda x.x)(\lambda x.a) \texttt{else} fail)]$   $\longrightarrow \ldots \longrightarrow a$  Call by Value  $\longrightarrow \ldots \longrightarrow fail$  Call by Name

### The labeled $\lambda$ -calculus (1/7)

Language

$$\begin{array}{c} \alpha, \beta, \gamma ::= \\ \mathbf{a} \mid \lceil \alpha \rceil \mid \lfloor \alpha \rfloor \mid \\ \alpha \beta \end{array}$$

 $\epsilon$  empty string

labels atomic name compound name

Exponent Rules

$$(M^{\alpha})^{\beta} = M^{\beta \alpha} \qquad M^{\epsilon} = M \qquad \lceil \epsilon \rceil = \lfloor \epsilon \rfloor = \epsilon$$

Reduction  $(\lambda x.M)^{\alpha}N \longrightarrow (M\{x := N^{\lfloor \alpha \rfloor}\})^{\lceil \alpha \rceil}$ 

$$\begin{aligned} \mathbf{x}^{\alpha} \{\mathbf{x} &:= \mathbf{P}\} &= \mathbf{P}^{\alpha} \\ \mathbf{y}^{\alpha} \{\mathbf{x} &:= \mathbf{P}\} &= \mathbf{y}^{\alpha} \\ (\lambda \mathbf{y} \cdot \mathbf{M})^{\alpha} \{\mathbf{x} &:= \mathbf{P}\} &= (\lambda \mathbf{y} \cdot \mathbf{M} \{\mathbf{x} &:= \mathbf{P}\})^{\alpha} \\ (\mathbf{M} \mathbf{N})^{\alpha} \{\mathbf{x} &:= \mathbf{P}\} &= (\mathbf{M} \{\mathbf{x} &:= \mathbf{P}\} \mathbf{N} \{\mathbf{x} &:= \mathbf{P}\})^{\alpha} \end{aligned}$$

#### The labeled $\lambda$ -calculus (2/7)

Graphically



M is sandwiched by [α] and [α]
 ⇒ theory of balanced paths [94, Asperti, Laneve, Guerrini, Mairson, Danos, Reigner, ...]
 ↔ Girard's geometry of interaction

## The labeled $\lambda$ -calculus (3/7)

- the labeled λ-calculus is confluent (thanks to exponent rules)
- the labeled λ-calculus tracks history of redexes (redex families)
- the labeled λ-calculus corresponds to the event structure of redexes
- ⇒ the labeled λ-calculus is a good candidate for a confluent equational theory of flow analysis (lattice of derivations, stability, ...)
   e.g. dependency calculus for *makefiles* uses a tiny subset

#### The labeled $\lambda$ -calculus (4/7)

- If M --> V, there is a unique minimum A of M such that
   A --> V [stability thm]
- If C[M] -- V, there is a unique minimum prefix A of M such that C[A] -- V' [corollary of stability thm]
- [97, Abadi, Lampson, JJL] compute minimum prefix by:
  - Mark all subexpression with different atomic label;
  - perform M→ V
  - erase part of *M* not in *V*.
- simple and good for incremental computations (Vista)
- also characterizes non interference when M = C[A]
   [99, Conchon, Pottier]

#### The labeled $\lambda$ -calculus (5/7)

• the labeled  $\lambda$ -calculus is good for tracing interactions.

 to build the Chinese Wall: Let *M* = *C*[*A*; *B*] → *V*. Let mark subexpressions in *A* with *a*, and in *B* with *b*. There should not be any label γ in *V* such that γ = ··· [*a*···*b*] ··· or γ = ··· [*a*···*b*] ···.

sets as labels

$$\begin{split} \llbracket \mathbf{a} \rrbracket_i &= \{\mathbf{a}\} \\ \llbracket \alpha \beta \rrbracket_i &= \llbracket \alpha \rrbracket_i \cup \llbracket \beta \rrbracket_i \\ \llbracket \lceil \alpha \rceil \rrbracket_1 &= \llbracket \lfloor \alpha \rfloor \rrbracket_1 = \{\llbracket \alpha \rrbracket_0\} \\ \llbracket \lceil \alpha \rceil \rrbracket_0 &= \llbracket \lfloor \alpha \rfloor \rrbracket_0 = \llbracket \alpha \rrbracket_0 \end{split}$$

where i = 0, 1 and  $\{\emptyset\} = \emptyset$ •  $\mathcal{P}(\alpha) = \neg \exists a \exists b. a, b \in X \in \llbracket \alpha \rrbracket_1$ 

- the labeled  $\lambda$ -calculus restricted by a predicate  $\mathcal{P}$ Reduction  $(\lambda x.M)^{\alpha}N \longrightarrow (M\{x := N^{\lfloor \alpha \rfloor}\})^{\lceil \alpha \rceil}$  when  $\models \mathcal{P}(\alpha)$
- the labeled λ-calculus restricted by P is still confluent for any P.

### The labeled $\lambda$ -calculus (7/7)

- Let  $\alpha < \beta$  be the causality relation:
  - $\begin{array}{l} \alpha < \lceil \alpha \rceil & \alpha < \lfloor \alpha \rfloor \\ \alpha < \beta \ \Rightarrow \ \alpha < \gamma \beta \delta \end{array}$
- Chinese Wall for independent spinoffs of A
   𝒫(α) = ¬(∃β ∃γ β ≤ γ ∧ γ ≤ β ∧ A < β < α ∧ A < γ < α)</li>
- β ≤ γ is not so easy to test equality between subtrees of the α tree
- simpler versions ? [Tomasz Blanc]
- from labeled λ-calculus towards DCC (Dependency Core Calculus) or other flow calculi with types ???
- deontic logic ?

#### Type systems and labels

[Sub]	
$\Gamma \vdash M : t$	$t \leq t'$
$\Gamma \vdash M$	: <i>t</i> ′

$$\frac{[Var]}{x \in domain(\Gamma)} \\ \frac{r \vdash x : \Gamma(x)}{\Gamma \vdash x : \Gamma(x)}$$

 $\frac{[Lambda]}{\Gamma, \mathbf{x} : t \vdash M : t'}$  $\frac{\Gamma \vdash \lambda \mathbf{x}.M : t \longrightarrow t'}{\Gamma \vdash \lambda \mathbf{x}.M : t \longrightarrow t'}$ 

$$\frac{[App]}{\Gamma \vdash M : t \xrightarrow{\alpha} t' \quad \Gamma \vdash N : \lfloor \alpha \rfloor \circ t} \frac{\Gamma \vdash M : [\alpha] \circ t}{\Gamma \vdash MN : \lceil \alpha \rceil \circ t'}$$

$$\frac{[Exponent]}{\Gamma \vdash M : t}$$
$$\frac{\Gamma \vdash M^{\alpha} : \alpha \circ t}{\Gamma \vdash M^{\alpha} : \alpha \circ t}$$

- pushing labels on types (with ≤)
- Infers [02, Pottier, Simonet]

- stack inspection is not static analysis
- dynamic checks support finer tests for security
- attempts for mixing history and stack inspection
- confluency is a hint for "good" calculi
  - stack inspection is not a good calculus
  - finer flow analysis
- statically scoped information (static permissions of stack inspection) should be carried by the labeled λ-calculus. (e.g. Chinese Wall)
- abstract interpretation of labeled lambda calculus?